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# Optimal Hedge Ratio estimation: GARCH (1,1) approach, a new model
# (Estimación óptima de Hedge Ratio: acercamiento de GARCH (1,1), un modelo nuevo)

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**Key words:** GARCH (1,1), hedge Ratio estimation

**Abstract.** An estimation of the Optimal Hedge Ratio on future markets is developed. The methodology incorporates forecasting the volatility and correlation of the spot and future prices using a GARCH (1,1) model, and under these estimations compute the optimal hedge ratio. This document shows a clear example of the methodology, using gold futures to hedge the risk exposure.

**Palabras Claves:** Estimación de Hedge Ratio, GARCH (1,1)

**Resumen.** Se desarrolla una estimación óptima de Hedge Ratio en los mercados del futuro. La metodología se basa en incorporar el pronostico de volatilidad y la correlación entre el precio del momento y del futuro usando el GARCH (1,1). Este documento demuestra un ejemplo claro de la metodología, utilizando el oro en el futuro para proteger el riesgo.

## Introduction

One of the most important problems for traders, investors and corporations interested in hedging their financial risk exposures, is how to determine the Optimal Hedge Ratio (*OHR*) of a certain underlying asset with financial futures. *OHR* by definition is the optimal ratio of the size of the position taken in future contracts to the size of the exposure. There are several research works on this topic. Some of them suggest calculating the *OHR* via the ratio of the covariance between spot and future price returns to the variance of the future prices (Benninga et al., 1983; Bell & Krasker, 1986; Hull, 2002). This is the equivalent to the slope of a simple regression of spot price changes and future price changes (Carter and Lyons; 1985). While other researchers have regressed spot price returns on future price returns, where returns are defined, as the proportional price change from period to period (Myers & Stanley, 1989).

The objective of this paper is to contribute to the literature with a methodology to determine an *OHR* in a statistically more significant manner than the models mentioned above. In the industry and in previous papers, the historical volatilities and correlation to compute the *OHR* are used. On the other hand we have found that the time series of the spot and future prices have an autocorrelation problem. Thus, the variables to compute the *OHR* are taken from the historical and auto correlated data.

The forecasted volatility and correlation are used in this document to compute the *OHR* appropriated. The projection is developed with a GARCH (1,1) and this methodology amends the autocorrelation trouble and incorporates a mean reversion property, since the model assumes stochastic volatility.

Thus, the question is: *Are the volatilities and correlation forecasted with the GARCH (1,1) model more appropriate to be computed by the OHR than the historical approach?*

We used the example of the Gold price hedging with futures for 66, 45, 30, 15 and 5 hedging days to demonstrate how the model works and how it is compared with the OLS estimation, we retrieved the historical information from Bloomberg, from Jul/15/1998 to Oct/26/2001. The methodology to forecast volatilities using the GARCH (1,1) model is based on the most recent observations of the returns; In addition to the most recent estimate of the variance rate, as well as on the GARCH parameters that maximize the probability occurrence of the variance rate (Castelino, 1992). To compute these parameters we used an approach called the maximum likelihood method (MLM). MLM's correlation forecasting is supported by the covariance projected which is calculated with the same assumption than the volatility (the mean reverting property). This method is explained in detail in section 2 of this paper.

The chief results of the model are that the longer the number of hedging days, the closer is the forecasted volatility, covariance and correlation to the long term ones, and giving a very similar result in the *OHR* as compared with the OLS methodology (Risk Metric, 1996). However, for short Term hedging periods, the difference in the results is distinct, with a considerable impact in the risk management policies of the hedgers. This essay concludes with the suggestion to use the GARCH (1, 1) methodology to calculate the *OHR* when financial futures are used.

## Methodology

We used an example of gold price hedging; considering financial futures (Bollerslev, 1986). Suppose for a gold mining company, the risk management strategy is to hedge the gold position exposure to the fluctuation of the gold prices, the use of the *OHR* for financial futures to employ the strategy was preferred. The information data used in this example was acquired from Bloomberg and includes daily gold spot prices as well as future gold prices of a contract with the cash settlement date on Dec/31/2001, from the period of Jul/15/1998 to Oct/26/2001 (Table 1). It is assumed hedging for 5, 15, 30, 45 and 66 days. With that information we computed the *OHR* using the following methodology.

*a) Computing daily returns and Long Term Variance rate*

First of all, we calculated the daily returns ($R_n$) of the both time series using the formula $\frac{P_n}{P_{n-1}}$ - 1, where $P_n$ is the most recent price and $P_{n-1}$ is the price from a period before. Next, we computed the daily Long Term Variance rate ($V_L$) of the both historical information, taking the variance of the historical returns. $V_L$ is important data for the calculation of the forecasted volatility. (Table 2 and 3, column (4) and (4A))

*b) Computing the daily variance ($\sigma_n^2$) of the returns using the GARCH (1, 1) model*

For computing the first variance rate, we used the formula $\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} R_{n-1}^2$ since it is an unbiased estimate of the variance rate per day using the most recent *m* observations of the returns. Next, we used the GARCH (1, 1) model to calculate the variance rate for the remaining observations. The equation for GARCH (1, 1) as it follows:

$$\sigma_n^2 = \gamma V_L + \alpha R_{n-1}^2 + \beta \sigma_{n-1}^2 \quad \textbf{(1)}$$

Where:

$\gamma$ = the weight assigned to $V_L$

$\alpha$ = the weight assigned to $R_{n-1}^2$

$\beta$ = the weight assigned to $\sigma_{n-1}^2$

The weights must sum to one $\gamma + \alpha + \beta = 1$

And $\omega = \gamma V_L$

So, the equation (1) can be rewritten as:

$\sigma_n^2 = \omega + \alpha R_{n-1}^2 + \beta \sigma_{n-1}^2$ **(2)**

Substituting for $\sigma_{n-1}^2$, would yield to:

$\sigma_n^2 = \omega + \alpha R_{n-1}^2 + \beta (\omega + \alpha R_{n-2}^2 + \beta \sigma_{n-2}^2)$

Continuing with this technique, we noted that the weight applied to $R_{n-i}^2$ is $\alpha\beta^{i-1}$, showing that the weights decline exponentially at a rate of $\beta$, making a function of a decay factor, and defines the relative importance of the observations on $R's$ in determining the current variance rate. The GARCH (1, 1) assigns weights that decline exponentially to past returns, as well as assigns some weight to the Long Term volatility (Danielsson, 1998).

In the next section it will be explained how to calculate the parameters $\gamma, \alpha, \beta$, but, for now we assigned arbitrary amounts.

The GARCH (1, 1) model assumes that the variances will be pulled back to the Long Term Variance Rate, the amount of weight for $V_L$ is $\gamma = 1 - \alpha - \beta$. So, this model considers how the volatility is driven by the following stochastic differential equation:

$dV = a(V_L - V)\ dt + \xi V dz$ **(3)**

Where:

a= 1-$\alpha + \beta$ ; it is time measured in days
$\xi$ = $\alpha\sqrt{2}$ ; mean reverting model

The variance has a drift that pulls it back to $V_L$ at rate of *a*. When *V*>$V_L$, the variance has a negative drift; when $V_L$ >*V* it has a positive drift (Nelson, 1990). Imposed on the drift is volatility $\xi$ (Table 2 and 3, column (5)) *c) Computing the parameters: Maximum Likelihood Approximation*

In order to calculate the GARCH parameters $\gamma, \alpha, \beta$ we utilized the historical data. The approach developed is the MLM. This method suggests values for the parameters that maximize the probability of the variance rate occurring.

We estimated the variance of the past returns, assuming the returns follow a normal distribution with zero mean and variance *v.* The likelihood of the returns observed is the value of the probability density function, which is represented by:

$$\frac{1}{\sqrt{2\pi v}} \exp(\frac{-R_i^{\ 2}}{2v})$$

The likelihood of the *m* observations occurring in the order in which they are observed is:

$$\prod_{i=1}^{m} [\frac{1}{\sqrt{2\pi v}} \exp(\frac{-R_i^{\ 2}}{2v})] \quad \textbf{(4)}$$

Under this approach, the best estimate of *v* is the value that maximizes this expression (Heynen & Kat, 1994). Maximizing an expression is the same to maximizing the logarithm. We calculated the logarithm and ignored constant multiplicative of the last expression having as a result:

$$\sum_{i=1}^{m} (-\ln(v) - \frac{R_i^2}{v}) = -m\ln(v) - \sum_{i=1}^{m} \frac{R_i^2}{v} \quad \textbf{(5)}$$

Now, differentiating with respect to *v* and setting the result equation to zero, the maximum likelihood estimator of *v* is:

$$\frac{\partial}{\partial v} - m\ln(v) - \sum_{i=1}^{m} \frac{R_i^2}{v} = 0$$

$$v = \frac{1}{m} \sum_{i=1}^{m} R_i^2$$

Now, we considered $v_i = \sigma_i^2$ as the variance estimated for day *i* and our assumption here is that the probability distribution of $R_i$ conditional on the variance is normal, and then we used the maximum likelihood approach which yields to:

$\prod_{i=1}^{m}[\frac{1}{\sqrt{2\pi v}}\exp(\frac{-R_i^{\,2}}{2v_i})$ **(6)**, then, taking logarithms we obtained:

$\sum_{i=1}^{m}(-\ln(v)-\frac{R_i^2}{v_i})$ **(7)**

The only difference with the equations (4) and (5) is the sub index *i* on the variance rate. Finally, an iteratively search was used to find the GARCH parameters (we used Solver of Microsoft Excel), which maximize the equation **(7)**. (Table 2 and 3, columns (6) and (6A)).

*d) Testing the autocorrelation before and after GARCH (1, 1)*

We tested the autocorrelation structure for the variables $R_i^2$ (before GARCH) and $\frac{R_i^2}{\sigma_i^2}$ (after GARCH) considering 15 lags and using the formula:

$\hat{\rho} = \frac{\sum_{i=k+1}^{T}\{(R_t-\overline{R})(R_{t-k}-\overline{R})/[T-(k-1)]}{\sum_{t=1}^{T}\{(R_t-\overline{R})^2\}/(T-1)}$ **(8)**, where:

*k* = Number of lags (days)

$\overline{R}$ = Average of the returns

$R_t$ = Return on day t

*T*= Number of observations

For testing the autocorrelation evidence, we used the Ljung-Box statistic, which explains that if a certain series has *m* observations, the statistic is:

$m\sum_{k=1}^{k}w_k\eta_k^2$, where:

$\eta_k$ = lag of *k*

$$w_k = \frac{m-2}{m-k}$$

For *k*=15, zero autocorrelation can be rejected with 95% confidence when Ljung-Box statistic is greater than 25. In table 3 and 4 the results show that the without GARCH model, there is a strong autocorrelation evidence. However, this problem has been corrected by the GARCH model in the spot prices time series as well as future price data. (Table 4 and 5).

*e) Forecasting Volatility Using GARCH (1, 1.)*

Plugging, $\gamma = 1 - \alpha - \beta$ in equation (1), the variance rate forecasted at day *n*-1, for day *n* is:

$$\sigma_n^2 = (1-\alpha-\beta)V_L + \alpha R_{n-1}^2 + \beta\sigma_{n-1}^2 \quad \textbf{(10)}$$

Then,

$$\sigma_n^2 - V_L = \alpha(R_{n-1}^2 - V_L) + \beta(\sigma_{n-1}^2 - V_L)$$

So, on day *n* + *k* in the future,

$$\sigma_{n+k}^2 - V_L = \alpha(R_{n+k-1}^2 - V_L) + \beta(\sigma_{n+k-1}^2 - V_L)$$

The expected value of $R_{n+k-1}^2$ is $\sigma_{n+k-1}^2$, therefore

$$E(\sigma_{n+k}^2 - V_L) = (\alpha+\beta)E(\sigma_{n+k-1}^2 - V_L)$$

Where *E* denotes the expected value, the equation repeatedly yields:

$$E(\sigma_{n+k}^2 - V_L) = (\alpha+\beta)^k(\sigma_n^2 - V_L) \text{ Or}$$

$$E(\sigma_{n+k}^2) = V_L(\alpha+\beta)^k(\sigma_n^2 - V_L) = \hat{\sigma} \quad \textbf{(11)}$$

This is the equation used to forecast the volatility utilized to compute the *OHR*. Once, the daily forecasted volatilities (square root of Expected variance rate) are computed for the total of hedging days, we calculated their average in order to plug in the *OHR* formula. These forecasted volatilities could be used to option pricing also (Tables 6 and 7).

*e) Forecasting Covariance and Correlations Using GARCH (1, 1).*

For forecasting the covariance between the spot and the future gold price returns, we used the formula:

$\hat{\text{cov}}_n = \omega + \alpha x_{n-1} y_{n-1} + \beta \, \text{cov}_{n-1}$ **(12)**, where:

$\omega, \alpha, \beta$ = Average of GARCH parameters of spot and future price returns

$x_{n-1}$ = Return of spot price on day *n-1*

$y_{n-1}$ = Return of future price on day *n-1*

$\text{cov}_{n-1}$ = covariance between spot and future price returns on day *n-1*

To compute the first covariance of the time series, we used the multiplication of the price returns between spot and future since is an unbiased estimate of the covariance, using the information available on that date.

To calculate the forecasting correlation, the following formula was used:

$$\hat{\rho}_{spot-future} = \frac{\hat{\text{cov}}_{spot-future}}{\hat{\sigma}_{spot} \, \hat{\sigma}_{future}}$$

(Table 8 and 9)

*f) Estimating OHR.*

After all calculations, we estimated the OHR for hedging the risk position with the formula:

$$OHR = \hat{\rho}_{spot-future} \frac{\hat{\sigma}_{spot}}{\hat{\sigma}_{future}} \quad \textbf{(13)}$$

That is the same as the coefficient of OLS, the only difference is the correlation and volatilities are forecasted from the GARCH model. Thus, they are more robust estimators. (Table 9)

## Results and conclusion

The results of the gold prices hedging showed in the table 9 are compared with the OLS results.

First of all, the OLS results are used for any period of days of hedging, where as the GARCH methodology has different results for each number of hedging days. The OLS considers the historical volatility (which is considered as the Long Term volatility in the GARCH model) to compute the *OHR* as well as the historical covariance and correlation between spot and future gold prices. Nothing like the GARCH model uses the forecasted estimators.

The most important findings from the results are, the longer the number of days of hedging, the closer the forecast volatility to the Long term volatility. This makes sense, since the projected volatility is driven by the stochastic differential equation (3) which has a mean reverting process. This suggests the longer the time, the projected volatility will be pulled back to the Long Term one. Therefore, in the calculation for the *OHR* in 66 days, the GARCH model expresses a very similar solution to the OLS, 40.57% of hedging versus 40.99%. Alternatively, the covariance and correlation forecast, has the same effect as the volatility, since, they are projected under the same mean reverting property assumption.

Nevertheless, for big quantities of hedging, this difference could be a significant high value for the hedger company. On the other hand, for shorter periods of hedging the difference is bigger, Thus, more relevant to the value creation driven by the risk management policy of the enterprises.

Another advantage of the GARCH model is that the time series are free from the autocorrelation problem. As a result, the OLS estimator shows important evidence of autocorrelation.

Our research suggests the use of the GARCH methodology for the calculation of the *OHR* for future periods. Nonetheless, in the long run, the results are very similar to those generated by the most common model (OLS). The reason for using this estimation methodology is basically, that the GARCH estimation is statistically more robust and there is a bigger difference in the results of hedging for short periods of time.

The model recommended here, works with any kind of financial assets and commodities. Therefore, the results should be very similar to the gold example presented in this paper. However, an extension of this research could be to calculate the *OHR* for an asset portfolio using a multivariate GARCH.

## Appendix

**Table 1**
**Historical Information**

| Date | Gold, Dec 2001 Future | Return | Gold Spot | Return |
|---|---|---|---|---|
| 15Jul1998 | 329.90 | | 293.15 | |
| 16Jul1998 | 329.70 | -0.0006 | 294.95 | 0.0061 |
| 17Jul1998 | 331.00 | 0.0039 | 293.80 | -0.0039 |
| - | - | - | - | - |
| - | - | - | - | - |
| - | - | - | - | - |
| 24Oct2001 | 276.30 | -0.0007 | 275.50 | -0.0020 |
| 25Oct2001 | 278.20 | 0.0069 | 276.60 | 0.0040 |
| 26Oct2001 | 278.30 | 0.0004 | 277.25 | 0.0023 |

**Table 2**
**Spot Gold Prices**

| (1) Date | (2) Day i | (3) Gold | (4) Return ($R_i$) | (5) $v_i = \sigma^2$ | (6) $-\ln(v_i) - R_i^2/v_i$ |
|---|---|---|---|---|---|
| 15Jul1998 | 1 | 293.15 | | | |
| 16Jul1998 | 2 | 294.95 | 0.0061 | | |
| 17Jul1998 | 3 | 293.8 | -0.0039 | 0.0000 | 9.7826 |
| 20Jul1998 | 4 | 294.85 | 0.0036 | 0.0000 | 9.7309 |
| 21Jul1998 | 5 | 294.9 | 0.0002 | 0.0000 | 9.9569 |
| - | - | - | - | - | - |
| - | - | - | - | - | - |
| - | - | - | - | - | - |
| 24Oct2001 | 824 | 275.5 | -0.0020 | 0.0001 | 9.5776 |
| 25Oct2001 | 825 | 276.6 | 0.0040 | 0.0001 | 9.5178 |
| 26Oct2001 | 826 | 277.25 | 0.0023 | 0.0001 | 9.7347 |
| | | | | | **6,977.1045** |

| | |
|---|---|
| σ | 0.0097 |
| VL | 0.0001 |

**(4A)**

**(6A)**
**GARCH parameters**

| | |
|---|---|
| **VL** | 0.0001 |
| **ω** | 0.0000 |
| **γ** | 0.2301 |
| **α** | 0.2775 |
| **β** | 0.4924 |
| **γ + α + β** | 1 |
| **α + β** | 0.7699 |

**Table 3**
**Future Gold Prices**

| (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|
| | | Gold, Dec 2001 | | | |
| Date | Day | Future | Return ($R_i$) | $v_i = \sigma^2$ | $-\ln(v_i) - R_i^2/v_i$ |
| 15Jul1998 | 1 | 329.9 | | | |
| 16Jul1998 | 2 | 329.7 | -0.0006 | | |
| 17Jul1998 | 3 | 331 | 0.0039 | 0.0000 | -27.4848 |
| 20Jul1998 | 4 | 332.7 | 0.0051 | 0.0000 | 9.5425 |
| 21Jul1998 | 5 | 331.8 | -0.0027 | 0.0000 | 9.9217 |
| - | - | - | - | - | - |
| - | - | - | - | - | - |
| - | - | - | - | - | - |
| 24Oct2001 | 824 | 276.3 | -0.0007 | 0.0001 | 9.3618 |
| 25Oct2001 | 825 | 278.2 | 0.0069 | 0.0001 | 8.9074 |
| 26Oct2001 | 826 | 278.3 | 0.0004 | 0.0001 | 9.6107 |
| | | | | | **7,006.6946** |

| | |
|---|---|
| σ | 0.0091684 |
| VL | 0.0000841 |

**(4A)**

**(6A)**
**GARCH parameters**

| | |
|---|---|
| **VL** | 0.0001 |
| **ω** | 0.0000 |
| **γ** | 0.2619 |
| **α** | 0.2190 |
| **β** | 0.5191 |

| | |
|---|---|
| **γ + α + β** | 1 |
| **α + β** | 0.7381 |

**Hedge Ratio estimation**

**Table 4**
**Spot Gold Prices**

| | Autocorrelation before GARCH | | Autocorrelation after GARCH | |
|---|---|---|---|---|
| Time Lag | for $R_i^2$ | w(lag) | for $R_i^2/\sigma_i^2$ | w(lag) |
| 1 | 0.19 | 1.00 | -0.01 | 1.00 |
| 2 | 0.06 | 1.00 | 0.00 | 1.00 |
| 3 | 0.07 | 1.00 | -0.01 | 1.00 |
| 4 | 0.06 | 1.00 | 0.00 | 1.00 |
| 5 | 0.08 | 1.00 | 0.00 | 1.00 |
| 6 | 0.04 | 1.00 | 0.01 | 1.00 |
| 7 | 0.04 | 1.01 | 0.00 | 1.01 |
| 8 | 0.03 | 1.01 | 0.01 | 1.01 |
| 9 | 0.01 | 1.01 | 0.00 | 1.01 |
| 10 | -0.01 | 1.01 | -0.01 | 1.01 |
| 11 | -0.01 | 1.01 | -0.01 | 1.01 |
| 12 | 0.00 | 1.01 | 0.00 | 1.01 |
| 13 | -0.01 | 1.01 | -0.01 | 1.01 |
| 14 | 0.00 | 1.01 | 0.00 | 1.01 |
| 15 | -0.01 | 1.02 | -0.01 | 1.02 |

**Ljung-Box**
**Before GARCH (1,1)**

| m | 825 |
|---|---|
| statistic | 49.39 |

**Ljung-Box**
**After GARCH (1,1)**

| m | 824 |
|---|---|
| statistic | 0.67 |

* If a certain Series has m observations, The Ljung-Box statistic is For lag=15, zero autocorrelation can be rejected with 95% confidence when the Ljung-Box statistic is greater than 25

* The GARCH (1,1) model has removed the autocorrelation
The GARCH (1,1) is working well

**Table 5**
**Future Gold Prices**

| Time Lag | Autocorrelation before GARCH for $R_i^2$ | w(lag) | Autocorrelation after GARCH for $R_i^2/\sigma_i^2$ | w(lag) |
|---|---|---|---|---|
| 1 | 0.20 | 1.00 | 0.01 | 1.00 |
| 2 | 0.02 | 1.00 | -0.01 | 1.00 |
| 3 | 0.05 | 1.00 | -0.02 | 1.00 |
| 4 | 0.19 | 1.00 | 0.02 | 1.00 |
| 5 | 0.11 | 1.00 | 0.02 | 1.00 |
| 6 | 0.11 | 1.00 | -0.02 | 1.00 |
| 7 | 0.04 | 1.01 | 0.02 | 1.01 |
| 8 | 0.00 | 1.01 | -0.01 | 1.01 |
| 9 | 0.00 | 1.01 | -0.01 | 1.01 |
| 10 | 0.00 | 1.01 | -0.01 | 1.01 |
| 11 | -0.01 | 1.01 | -0.01 | 1.01 |
| 12 | 0.02 | 1.01 | 0.02 | 1.01 |
| 13 | 0.01 | 1.01 | 0.03 | 1.01 |
| 14 | 0.03 | 1.01 | 0.03 | 1.01 |
| 15 | 0.00 | 1.02 | -0.02 | 1.02 |

**Ljung-Box Before GARCH (1,1)**

| | |
|---|---|
| **m** | **825** |
| **statistic** | **86.05** |

**Ljung-Box After GARCH (1,1)**

| | |
|---|---|
| **m** | **824** |
| **statistic** | **4.30** |

* If a certain Series has m observations, The Ljung-Box statistic is For lag=15, zero autocorrelation can be rejected with 95% confidence when the Ljung-Box statistic is greater than 25

* The GARCH (1,1) model has removed the autocorrelation The GARCH (1,1) is working well

**Hedge Ratio estimation**

**Table 6**
**Forecasting Volatility Spot Gold Prices**

| | |
|---|---|
| **Friday, October 26, 2001** | 0.0001 |
| **VL** | 0.0001 |
| **alfa+beta** | 0.7381 |

| Date | day (k) | $\sigma_i^2$ Forcasted |
|---|---|---|
| Saturday, October 27, 2001 | 1 | 0.000064 |
| Sunday, October 28, 2001 | 2 | 0.000078 |
| Monday, October 29, 2001 | 3 | 0.000088 |
| - | - | - |
| - | - | - |
| - | - | - |
| Saturday, December 29, 2001 | 64 | 0.000095 |
| Sunday, December 30, 2001 | 65 | 0.000095 |
| Monday, December 31, 2001 | 66 | 0.000095 |

| | |
|---|---|
| **$\sigma_i^2$ Forcasted** | 0.000094 |
| **$\sigma_i$ Forcasted (daily)** | 0.97% |
| **$\sigma_i$ Forcasted (annual)** | **15.40%** |

**Table 7**
**Forecasting Volatility Future Gold Prices**

| | |
|---|---|
| **Friday, October 26, 2001** | 0.0001 |
| **VL** | 0.0001 |
| **alfa+beta** | 0.7381 |

| Date | day (k) | $\sigma_i^2$ Forcasted |
|---|---|---|
| Saturday, October 27, 2001 | 1 | 0.00007 |
| Sunday, October 28, 2001 | 2 | 0.00008 |
| Monday, October 29, 2001 | 3 | 0.00008 |
| - | - | - |
| - | - | - |
| - | - | - |
| Sunday, December 30, 2001 | 65 | 0.00008 |
| Monday, December 31, 2001 | 66 | 0.00008 |
| | | 0.00008 |

| | |
|---|---|
| **$\sigma_i^2$ Forcasted** | 0.0001 |
| **$\sigma_i$ Forcasted (daily)** | 0.92% |
| **$\sigma_i$ Forcasted (annual)** | **14.55%** |

**Table 8**
**Forecasting Covariance Spot-Future Gold Prices**

| Date | day (k) | cov ( spot, future) forcasted |
|---|---|---|
| Saturday, October 27, 2001 | 1 | 0.0054 |
| Sunday, October 28, 2001 | 2 | 0.0068 |
| Monday, October 29, 2001 | 3 | 0.0079 |
| - | - | - |
| - | - | - |
| - | - | - |
| Saturday, December 29, 2001 | 64 | 0.0087 |
| Sunday, December 30, 2001 | 65 | 0.0087 |
| Monday, December 31, 2001 | 66 | 0.0087 |

| | |
|---|---|
| **cov ( spot, future) forcasted** | **0.0086** |

**Table 9**
**Optimal Hedge Ratio**

**GARCH (1,1) estimator**

| Hedging Days | 66 | 45 | 30 | 15 | 5 |
|---|---|---|---|---|---|
| Forcasted σ spot annual | 15.40% | 15.37% | 15.31% | 15.16% | 14.52% |
| Forcasted σ future annual | 14.55% | 14.51% | 14.49% | 14.42% | 14.14% |
| Forcasted cov ( spot, future) | 0.0086 | 0.0085 | 0.0085 | 0.0083 | 0.0074 |
| Forcasted ρ spot-futures | 0.38 | 0.38 | 0.38 | 0.38 | 0.36 |
| **Optimal Hedge Ratio** | **40.57%** | **40.58%** | **40.37%** | **39.75%** | **37.14%** |

**OLS estimator**

| Any amount of hedging days | |
|---|---|
| ρ spot-futures | 0.39 |
| σ spot daily | 0.97% |
| σ spot annual | **15.47%** |
| σ future daily | 0.92% |
| σ future annual | **14.55%** |
| cov ( spot, future) | 0.0087 |
| **Optimal Hedge Ratio** | **40.99%** |

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